SAT/ACT MATH

Formula Sheet

Algebra

1.Order of operation PEMDAS: -

(Parentheses → exponents → Multiply →divide → add → subtract).

2.Factoring & solving: -

$(x + a)(x + b) = x^2 + (a + b)x + ab$
$a^2 - b^2 = (a + b)(a - b)$
$(a \pm b)^2 = a^2 + b^2 \pm 2ab$

3.Percentages: -

Part = $\frac{percent}{100}$ × whole
Percentage change=$\frac{new-old}{old}$ × 100%

4.Rate: -

Speed = $\frac{Distance}{time}$ (Frequently used formula in rate)

5.Exponents & radicals: -

$x^a \cdot x^b = x^{a+b}$
$(x^a)^b = x^{ab}$
$x^a/x^b = x^{a-b}$
$(xy)^a = x^a \cdot y^a$
$1/x^a = x$ - a
$\sqrt[a]{xy} = \sqrt[a]{x}\sqrt[a]{y}$

$x^a = y \rightarrow x = y^{1/a}$

$(-1)^n = \{+1 \ if\ n\ is\ even \ -1 \ if\ n\ is\ odd$

6.Arithmetic Sequence: -

n^{th} term = a_n= a+(n-1)d
Sum of n term = S_n = $\frac{n}{2}$(2a+ (n-1)d)
Where a = first term, d= common difference

7.Geometric Sequence: -

n^{th} term = a_n= $a \cdot r^{n-1}$
Sum of n term = S_n = $\frac{a(r^n-1)}{r-1}$
Where a = first term, r= common ratio

8.Numbers: -

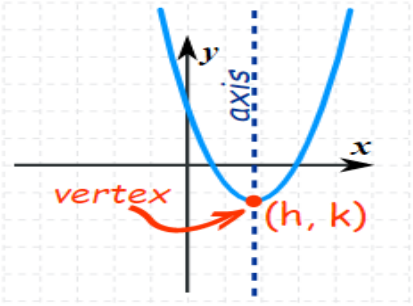
Sum of consecutive integers from 1 to n = $\frac{n(n+1)}{2}$

$\frac{a}{b}$ is undefined when denominator (b) = 0
as a increases, $\frac{a}{b}$ also increases (keeping b same)
as b increases, $\frac{a}{b}$ decreases (keeping a same)
if $\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$ (cross multiplication)

9.Quadratic equations: -

$$y = ax^2 + bx + c$$

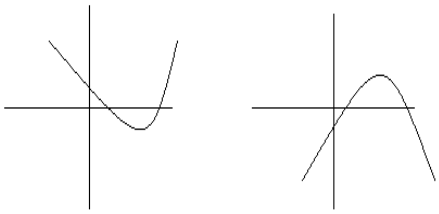
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$



Sum of roots = $\frac{-b}{a}$
Product of roots = $\frac{c}{a}$
$h = \frac{-b}{2a}$ = average of two roots
D = Discriminant = $b^2 - 4ac$

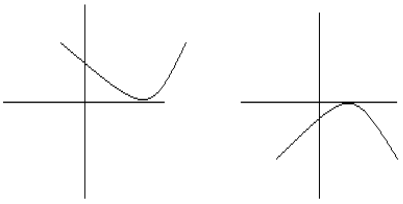
D > 0- Two distinct real roots

a > 0 a < 0



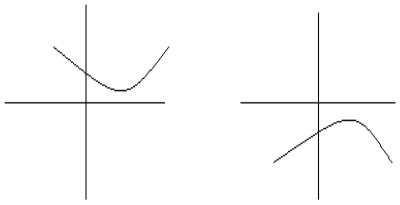
D = 0 -equal roots

a > 0 a < 0



D < 0- Imaginary roots

α > 0 α < 0



Vertex form for y = $ax^2 + bx$ +c

$y = \alpha(x - h)^2 + k$

10.Linear inequalities: -

α < x < b

α $\pm$ c < x $\pm$ c < b $\pm$ c

αc < (x)(c) < bc (of c > 0)

bc < (x)(c) < αc (if c < 0)

α > b → $\frac{1}{a} < \frac{1}{b}$

11.Absolute Value: -

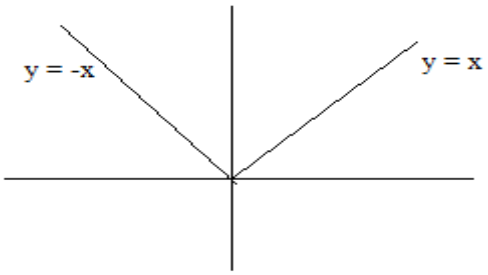
If |x| = 5, then x = 5 or -5

If |x| > 5, then x > 5 or x < -5

If |x| < 5, then -5 < x < 5

If |x| = α (α < 0), then no solution for x

$y = |x| = \{x, \ if \quad x \geq 0 \ , -x \ \ if \quad x < 0$



y=|x|

12.Functions: -

y =f(x)

f: x → y

Input (x) → $f(x)$ → utput (y)

Set of all possible inputs is Domain

Set of all possible outputs is Range

Transformation of function - y =f(x)

	Nature	Change
$f(x)+\alpha$	α > 0	Moves vertically (↑) up
$f(x)+\alpha$	α < 0	Moves vertically (↓) down
$f(x+\alpha)$	α > 0	Moves left (←)
$f(x+\alpha)$	α < 0	Moves right (→)
$\alpha f(x)$	α > 0	Vertical dilation by scale α
$f(\alpha x)$	α > 0	Horizontal dilation by scale α
$-f(x)$	-	Reflection about x - axis
$f(-x)$	-	Refelection about y-axis
$\|f(x)\|$	-	Reflection of negative 'y' part about x - axis

13.Complex numbers: -

i = $\sqrt{-1}$

i^2 = 1

i^3= - 1

i^4 = 1

α ± bi are conjugate pairs

14.Logarithms: -

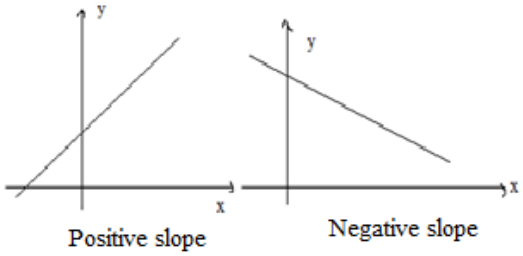
$log_a x$ = n is equivalent to $a^n = x$

$$log_a x + log_a y = log_a xy$$
$$log_a x - log_a y = log_a \frac{x}{y}$$

$log_a x = \frac{log_c x}{log_c a}$

15.Growth and Decay: -

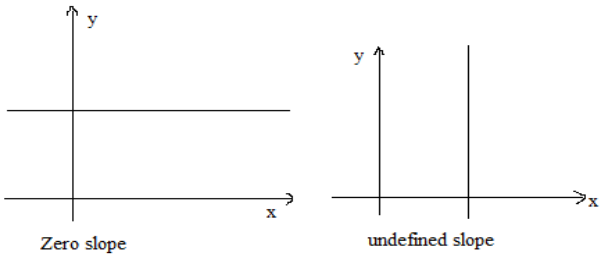
General form – A = P (1± r)^t
Exponential form – A = Pa^{rt}
Compounding form – A = P (1 ± $\frac{r}{n}$)^{nt}



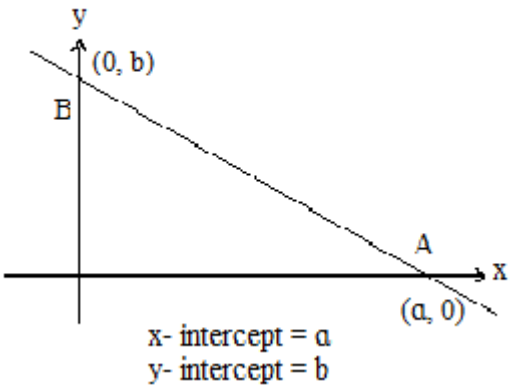
Statistics and probability: -

Mean = $\frac{sum\ of\ terms}{Number\ of\ terms(n)}$
Mode – Most frequent term
Median – Middle value in the list
If n is odd → Median is ($\frac{n+1}{2}$)th value
If n is even → Median is average of

value at ($\frac{n^{th}}{2}$) and ($\frac{n}{2}+1$)th position.



Probability = $\frac{number\ of\ desired\ outcome}{number\ of\ total\ outcome}$
P (A and) = P(A)·P(B) if events A and B are independent
P (A or B) = P(A) + P(B) – P (A & B)
Conditional probability: P (A | B) = $\frac{P(A\ \&\ B)}{P(B)}$



Permutation and Combination: -

Counting principle –
N ways for event A
M ways for event B
Then NXM ways for event A and B together number of permutations of n things taken r at a time is
P (n, r) = $\frac{n!}{(n-1)!}$
Number of combinations of n things taken r at a time is
C (n, r) = $\frac{n!}{(n-r)!\cdot r!}$

Coordinate Geometry: -

A (x_1, y_1) and (x_2, y_2)

Distance AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of $\overline{AB}$ = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope of line AB(m) = $\frac{y_2-y_1}{x_2-x_1}$

Line equation: -

Point slope form (y- y_1) = m(x- x_1)
Slope intercept form y= mx + c
(c = y- intercept)
For parallel lines: $m_1 = m_2$ (equal slopes)
For perpendicular lines: $m_1 \times m_2$ = -1

System of equations: -

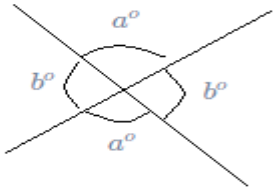
$a_1x + b_1y = c_1$

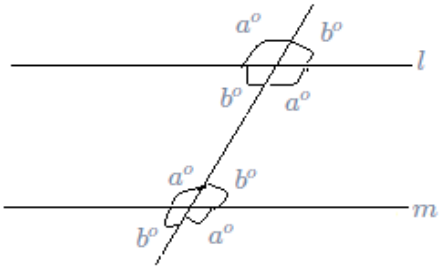
$a_2x + b_2y = c_2$

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ → No solution
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ → Infinite solutions

Geometry: -

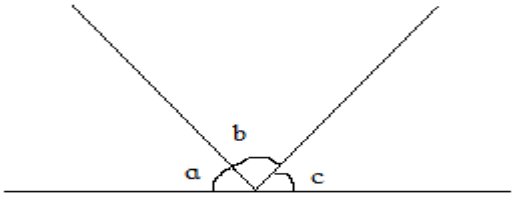
1. Angles





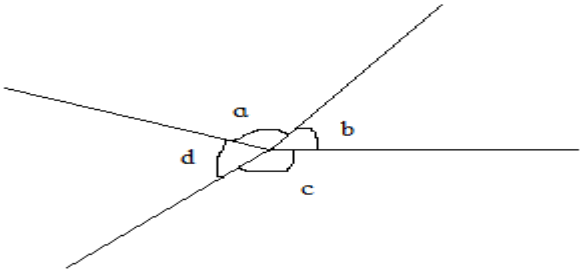
l and m are parallel

sum of any number of angles forming a straight line is 180°



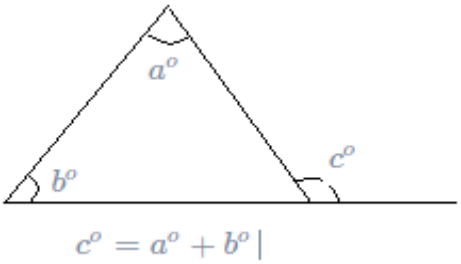
$a + b + c = 180°$

Sum of any number of angles around a point is 360°



$a + b + c + d = 360°$

If ∠a + ∠b = 90°, then a and b complementary angles.
If ∠a + ∠b = 180°, then a and b are supplementary angles.

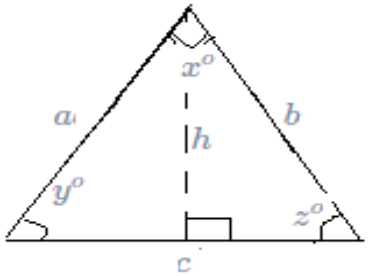


$c^o = a^o + b^o$

2. Polygon (n = number of sides): -

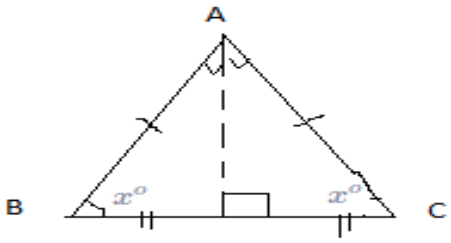
Sum of all interior angles = 180°(n-2)
Sum of all exterior angles= 360°

Triangles: -

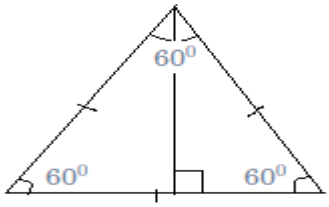


a + b > c |a-b|<c
b + c > a |b-c|<a
c + a > b |c-a|<b

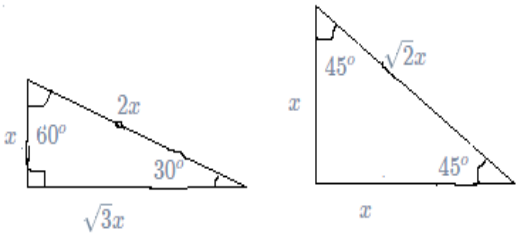
x° + y° + z° = 180°
Area = $\frac{1}{2}$ × b × h
Side opposite to largest angle is largest and vice versa
Isosceles –



Equilateral –



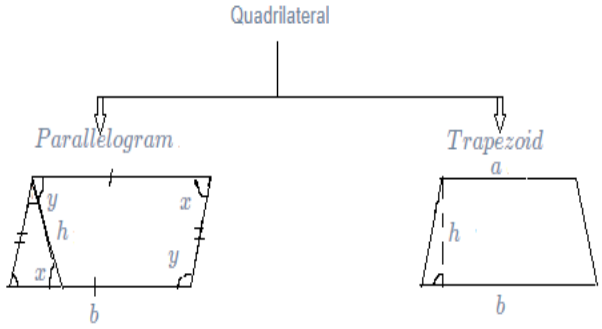
Right angle-



Pythagoras theorem- $a^2+b^2=c^2$

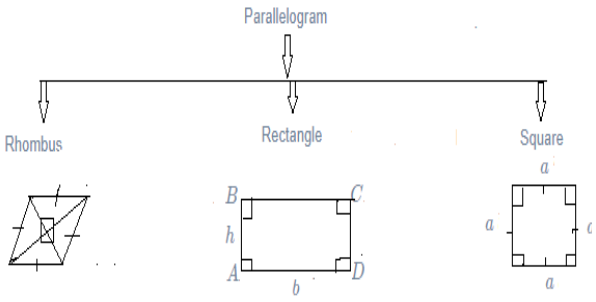
(Where, c is hypotenuse)

Pythagorean triplets-(3,4,5),(12,5,13),(6,8,10)



- opposit sides are parallel
- x + y = 180°
- Diagonals bisect each other
- Area = $base(b) \times height(h)$

- one pair of parallel sides
- Area = $\frac{1}{2} \times h \times (a+b)$



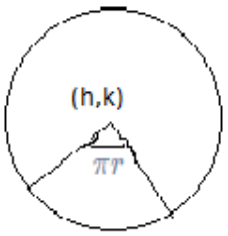
- All sides are equal
- Diagonals bisect each other at right angle
- Area = $\frac{1}{2} \times (d1) \times (d2)$

d1 and d2 are length of diagonls

- AC = BD
- Area $= b \times h$

- Area $= a^2$

3.Circle: -

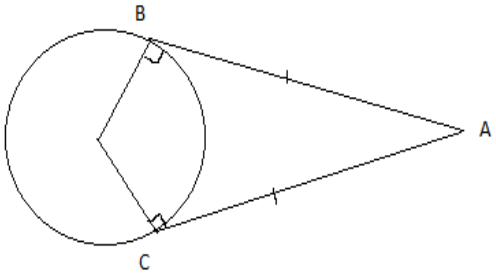


Area = $лr^2$

Circumference = 2лr

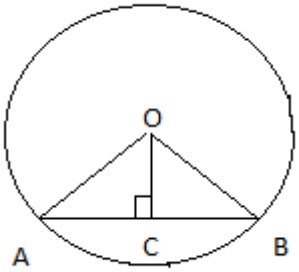
Length of arc = $\left(\frac{n°}{360°}\right)$ 2лr

Area of sector = $\left(\frac{n°}{360°}\right) \times лr^2$



AB = AC

Tangent and radius are at right angle.



AC = BC

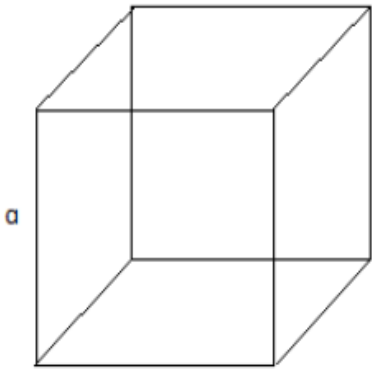
AB is chord

Equation of circle- $(x-h)^2+(y-k)^2=r^2$

Radian = degree × $\left(\frac{л}{180°}\right)$

4.Solid figure: -

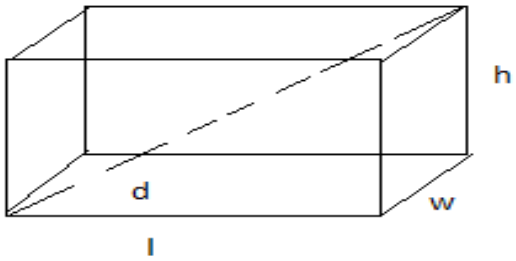
Cube



V = a^3

SA= $6a^2$

Rectangular box



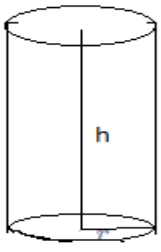
V = l × w × h

d = $\sqrt{l^2 + w^2 + h^2}$

surface area (SA)

= 2(lw + wh + hl)

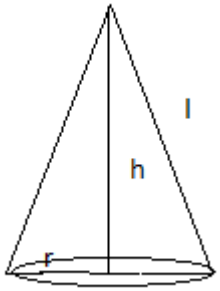
Right circular cylinder



V = $\pi r^2 h$

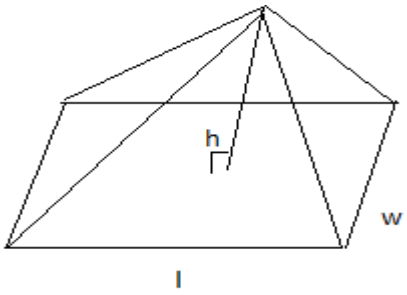
SA = $2\pi r^2$ + 2πrh

Right Circular cone



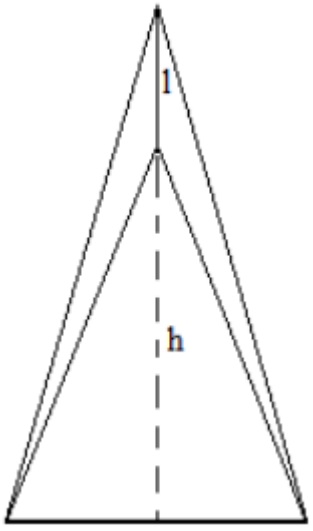
V = $\frac{1}{3}\pi r^2 h$

Pyramid with rectangular base



V = $\frac{1}{3}$ × l × w × h

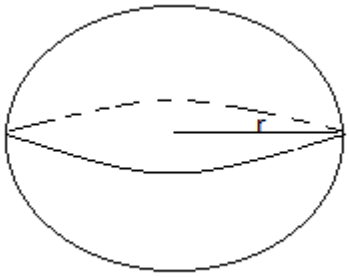
Triangular Pyramid



V= $\frac{1}{3}$ × A × h

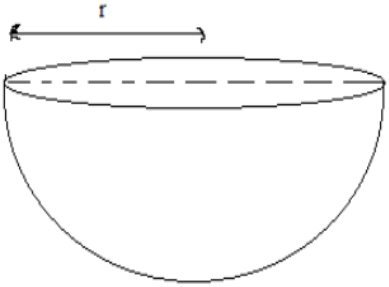
Where,A = area of triangular base

Sphere



$V = \frac{4}{3} \pi r^3$

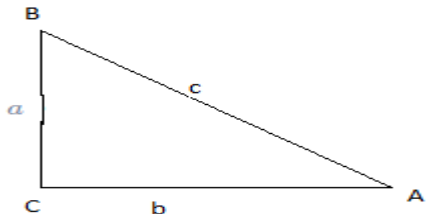
$SA = 4\pi r^2$



$V = \frac{2}{3} \pi r^3$

$SA = 3\pi r^2$

Trigonometry: -



Sin A = $\frac{a}{c}$

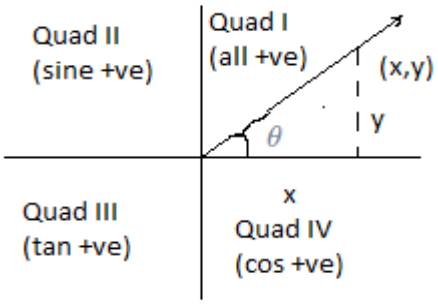
Cos A = $\frac{b}{c}$

Tan A = $\frac{a}{b}$

$\frac{Sin\ A}{Cos\ A}$ = tan A

Sin A = Cos (90 – A) =CosB

Unit circle



Sin θ = $\frac{y}{r}$

Cos θ = $\frac{x}{r}$

Tan θ = $\frac{y}{x}$

Sin (x ±360°) = sin x

Cos (x ±360°) = cos x

Tan (x ±360°) = tan x

Csc θ = $\frac{1}{sin\ \theta}$

Sec θ = $\frac{1}{cos\ \theta}$

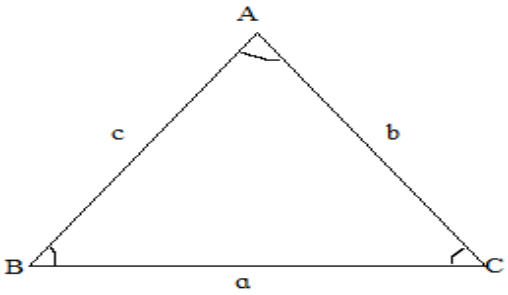
Cot θ = $\frac{1}{tan\ \theta}$

Identities

$Sin^2\theta + Cos^2\theta = 1$

$1 + tan^2\theta = sec^2\theta$

$1 + cot^2\theta = csc^2\theta$



Sine rule-

$\frac{sin\ A}{a} = \frac{sin\ B}{b} = \frac{sin\ C}{c}$

Cosine rule

Cos A = $\frac{b^2+c^2-a^2}{2bc}$

Cos B = $\frac{a^2+c^2-b^2}{2ac}$

Cos C = $\frac{a^2+b^2-c^2}{2ab}$